-continued

$$\hat{\sigma}_{s_p}^2(z_{t+h}) = \frac{\left(1.\hat{\mu}_h(z_{t+h}) - \Delta Y_{t,s_p}\right)^T R_{s_p}\left(Z_{t,s_p}, Z_{t,s_p}; \theta\right)^{-1}\left(1.\hat{\mu}_h(z_{t+h}) - \Delta Y_{t,s_p}\right)}{m}$$

$$R_{s_p}\left(Z_{t,s_p}(i), Z_{t,s_p}(j); \theta\right) = \exp\left(-\sum_{l=1}^{n} \theta_l (z_{il} - z_{jl})^{q_l}\right)$$

[0075] In some embodiments, a modified Cholesky decomposition may be used for the correlation matrix $R_{s_p}(\theta)=C_{s_p}(\theta)C_{s_p}{}^t(\theta)$ with q set at 1. For the value of θ, the optimal value may be maximized using log-likelihood using a pattern search algorithm, for example, using the equation:

$$J_{s_p}(z_{t+h}; \theta) = \frac{-\left(m \log\hat{\sigma}_{s_p}^2(z_{t+h}) + \log\left|R_{s_p}\left(Z_{t,s_p}, Z_{t,s_p}; \theta\right)\right|\right)}{2}$$

or minimized using the equation:

$$J_{s_p}(z_{t+h};\theta)=\hat{\sigma}_{s_p}{}^2(z_{t+h})det(C(\theta))^{2/m}$$

where

$$\hat{\sigma}_{s_p}^2(z_{t+h}) = \frac{\left(1.\hat{\mu}_h(z_{t+h}) - \Delta Y_{t,s_p}\right)^T R_{s_p}\left(Z_{t,s_p}, Z_{t,s_p}; \theta\right)^{-1}\left(1.\hat{\mu}_h(z_{t+h}) - \Delta Y_{t,s_p}\right)}{m}$$

[0076] Using these values, a prediction for the error forecast $\hat{e}_{t+h}$ and MSE may be calculated as

$$\begin{aligned} MSE(\hat{e}_t) &= \mathbb{E}[(\hat{e}_t - e_t)^2] \\ &= \mathbb{E}[(\hat{e}_t - \mathbb{E}[\hat{e}_t])^2] + (\mathbb{E}(\hat{e}_t - e_t))^2 \\ &= Var(\hat{e}_t) + Bias(\hat{e}_t, e_t)^2 \end{aligned}$$

where the probability distribution of the forecast error may be computed as

$$e_{t+h}=y_{t+h}-\hat{y}_{y+H}$$

$$\hat{e}_{t+h}=N(\mu_h,\sigma_h)$$

$$y_{t+h}=\hat{y}_{t+h}+N(\mu_h, \sigma_h)$$

where e_{t+h} may represent the real value for error, $\hat{e}y_{t+h}$ may represent the error distribution calculated in accordance with the present disclosure, Using these calculations, the expected value and variance of the error for the forecast may be determined, for example, by the equations

$$\sigma_h=\sqrt{MSE(\hat{e}_{t+h})-\text{Bias}(\hat{e}_{t+h},e_{t+h})^2}$$

$$\mu_h=\hat{e}_{t+H}$$

$$y_{t+h}=\hat{y}_{t+h}+N(\mu_h, \sigma_h)y_{t+h}=\hat{y}_{t+h}+\hat{e}_{t+h}$$

where μ_h may represent the mean value of error, and σ_h may represent variance associated with the error, and a distribution of error, $\hat{e}_{t+h}$ may be obtained.

[0077] As used in the present disclosure, the terms "module" or "component" may refer to specific hardware implementations configured to perform the actions of the module or component and/or software objects or software routines that may be stored on and/or executed by general purpose hardware (e.g., computer-readable media, processing devices, etc.) of the computing system. In some embodiments, the different components, modules, engines, and services described in the present disclosure may be implemented as objects or processes that execute on the computing system (e.g., as separate threads). While some of the system and methods described in the present disclosure are generally described as being implemented in software (stored on and/or executed by general purpose hardware), specific hardware implementations or a combination of software and specific hardware implementations are also possible and contemplated. In this description, a "computing entity" may be any computing system as previously defined in the present disclosure, or any module or combination of modulates running on a computing system.

[0078] Terms used in the present disclosure and especially in the appended claims (e.g., bodies of the appended claims) are generally intended as "open" terms (e.g., the term "including" should be interpreted as "including, but not limited to," the term "having" should be interpreted as "having at least," the term "includes" should be interpreted as "includes, but is not limited to," the term "containing" should be interpreted as "containing, but not limited to," etc.).

[0079] Additionally, if a specific number of an introduced claim recitation is intended, such an intent will be explicitly recited in the claim, and in the absence of such recitation no such intent is present. For example, as an aid to understanding, the following appended claims may contain usage of the introductory phrases "at least one" and "one or more" to introduce claim recitations. However, the use of such phrases should not be construed to imply that the introduction of a claim recitation by the indefinite articles "a" or "an" limits any particular claim containing such introduced claim recitation to embodiments containing only one such recitation, even when the same claim includes the introductory phrases one or more or at least one and indefinite articles such as "a" or "an" (e.g., "a" and/or "an" should be interpreted to mean "at least one" or "one or more"); the same holds true for the use of definite articles used to introduce claim recitations.

[0080] In addition, even if a specific number of an introduced claim recitation is explicitly recited, those skilled in the art will recognize that such recitation should be interpreted to mean at least the recited number (e.g., the bare recitation of "two recitations," without other modifiers, means at least two recitations, or two or more recitations). Furthermore, in those instances where a convention analogous to "at least one of A, B, and C, etc." or "one or more of A, B, and C, etc." is used, in general such a construction is intended to include A alone, B alone, C alone, A and B together, A and C together, B and C together, or A, B, and C together, etc.

[0081] Further, any disjunctive word or phrase presenting two or more alternative terms, whether in the description, claims, or drawings, should be understood to contemplate the possibilities of including one of the terms, either of the terms, or both terms. For example, the phrase "A or B" should be understood to include the possibilities of "A" or "B" or "A and B."

[0082] All examples and conditional language recited in the present disclosure are intended for pedagogical objects to aid the reader in understanding the disclosure and the